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LETTER TO THE EDITOR

ON SLOSHING IN A CONTAINER WITH MOVING WALL

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The research on the coupled frequencies of a fluid–structure system comprised of a container with a moving wall partially filled with water (Figure 1) was presented in two papers by Lu *et al.* and Chai *et al.*, but their solutions are different. The aim of this letter is to compare them. The fluid is incompressible and inviscid, and the structure is a mass m [$\mathrm{kg\,m^{-1}}$] in translation, connected to the Galilean reference by a spring of stiffness k [$\mathrm{N\,m^{-2}}$]; these characteristics are given per unit length in the z direction. The authors linearized the equations and looked for a potential-flow solution for the fluid motion. They obtain the same set of equations.

1. SOLUTION OF CHAI ET AL.

CHAI *ET AL.* (1996) GAVE a dimensionless form of the equations, using the length L of the container, the gravity g, and the mass of the contained fluid ρhL [$\mathrm{kg\,m^{-1}}$], with ρ being the fluid density [$\mathrm{kg\,m^{-3}}$]. The dimensionless pulsation $\bar{\omega}$ is given by $\bar{\omega} = \omega\sqrt{L/g}$, the position $\bar{x} = x/L$, the fluid depth $\bar{h} = h/L$, the time $\bar{t} = t\sqrt{g/L}$, the potential $\bar{\phi} = \phi/(L\sqrt{Lg})$, the mass $\bar{m} = m/(\rho hL)$, the stiffness of the spring $\bar{k} = k/(\rho gh)$ (Figure 1).

Chai *et al.* (1996) immediately introduced a periodic solution,

$$\bar{w}(\bar{t}) = c_1 \cos(\bar{\omega}\bar{t}), \tag{1}$$

$$\bar{\phi}(\bar{x}, \bar{y}, \bar{t}) = \left(\beta \frac{\cos[q(\bar{x}-1)]}{\sin(q)} \cosh(q\bar{y}) + \sum_{n=0}^{\infty} \alpha_n \frac{\cosh[p_n(\bar{x}-1)]}{\sinh(p_n)} \cos(p_n\bar{y})\right) \bar{\omega} \sin(\omega\bar{t}), \tag{2}$$

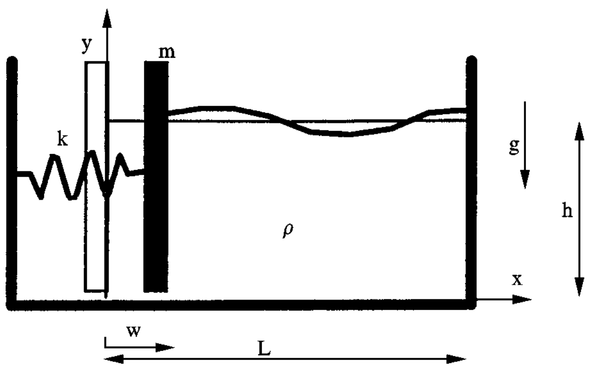

Figure 1. A container with a moving wall (translation) partially filled with a fluid.

with $\bar{\omega}$ the dimensionless coupled circular frequency. The parameters q and p_n are the real positive roots of the transcendental equations

$$\bar{\omega}^2 = -\, p_n \tan(p_n \bar{h}), \tag{3}$$

$$\bar{\omega}^2 = q \tanh(q\bar{h}), \tag{4}$$

and the parameters α_n and β are proportional to the amplitude of the piston-type motion of the wall c_1:

$$\alpha_n = \frac{-\,4 \sin(p_n \bar{h})}{p_n(2p_n\bar{h} + \sin(2p_n\bar{h}))}\, c_1, \tag{5}$$

$$\beta = \frac{4 \sin(q\bar{h})}{q(2q\bar{h} + \sinh(2q\bar{h}))}\, c_1, \tag{6}$$

The coupled circular frequency is obtained looking for the $\omega = \omega_{c\mathrm{Chai}}$ for which the dynamic equation of the structure is satisfied,

$$\bar{m}\frac{\mathrm{d}^2\bar{w}}{\mathrm{d}\bar{t}^2} + \bar{k}\bar{w} = -\int_{\bar{y}=0}^{\bar{h}} \bar{\rho}\,\frac{\partial\bar{\phi}}{\partial\bar{t}}\,\mathrm{d}\bar{y}. \tag{7}$$

2. SOLUTION OF LU ET AL.

In contrast, Lu *et al.* (1997) supply the unstationary response of the fluid to the motion of the wall $w(t) = \xi(t)$. The velocity potential is taken as

$$\phi(x, y, t) = \frac{x^2 - y^2}{2L}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \sum_{n=1}^{\infty} \frac{\beta_n}{\omega_n \cosh(k_n h)} \cos(k_n x)\cosh(k_n y) \int_0^t \frac{\mathrm{d}^3\xi}{\mathrm{d}\tau^3} \cos[\omega_n(t-\tau)]\,\mathrm{d}\tau, \tag{8}$$

with $\omega_n = \sqrt{gh\tanh(k_n h)}$ being the sloshing circular frequencies in a rigid container, $k_n = n\pi/L$ and $\beta_n = 2(-1)^n/k_n L$. The dynamic equation is obtained as

$$(m + E)\frac{\mathrm{d}^2\xi}{\mathrm{d}t^2} + k\xi = \sum_{n=0}^{\infty} F_n \int_0^t \frac{\mathrm{d}^3\xi}{\mathrm{d}\tau^3}\cos[\omega_n(t-\tau)]\,\mathrm{d}\tau, \tag{9}$$

with $E = \rho(h/L)(L^2/2 - (h^2/6))$, $F_n = (2\rho L^2)/(n\pi)^3)\tanh(k_n h)$. A Laplace transform is then used to obtain the value of $\xi(s)$. The zeros of the denominator give the coupled frequencies, so the pulsation ω must satisfy

$$(m + E)s^2 + k - \sum_{n=0}^{\infty} F_n \frac{s^4}{s^2 + \omega_n^2} = 0. \tag{10}$$

with $s^2 = -\omega_{c\,\mathrm{Lu}}^2$.

3. COMPARISON OF THE SOLUTIONS

The comparison is made in the dimensionless forms. Figure 2 presents the evolution of the coupled frequencies $\bar{\omega}_{c\,\mathrm{Chai}}$ and $\bar{\omega}_{c\,\mathrm{Lu}}$, the frequency of the dry structure $\bar{\omega}_d$, the frequency of the coupled structure with only the added mass effect $(g = 0)\bar{\omega}_m$, the sloshing frequencies in a rigid container of the same dimensions $\bar{\omega}_s$, with respect to the dimensionless stiffness $\bar{k}$, for $\bar{m} = 1$, $\bar{h} = 0{\cdot}4$.

We notice that the evolutions of the two models are similar: a coupled frequency exists between two sloshing frequencies. These solutions are very different from the frequency of the dry structure ($\bar{\omega}_d$), and the coupled frequency with only added mass affect ($\bar{\omega}_m$). When $\bar{k} \to \infty$ the two models predict the sloshing frequencies. When $\bar{k} \to 0$, the coupled frequencies decrease towards a sloshing frequency. The two solutions present some differences, however: (i) when $\bar{k}$ is increased, the evolution of the Chai *et al.* solution more abrupt than the Lu *et al.* solution; (ii) when $\bar{k} \to 0$, the Lu *et al.* coupled frequencies do not seem to approach the sloshing frequency.

Experiments will determine which model may be the more correct.

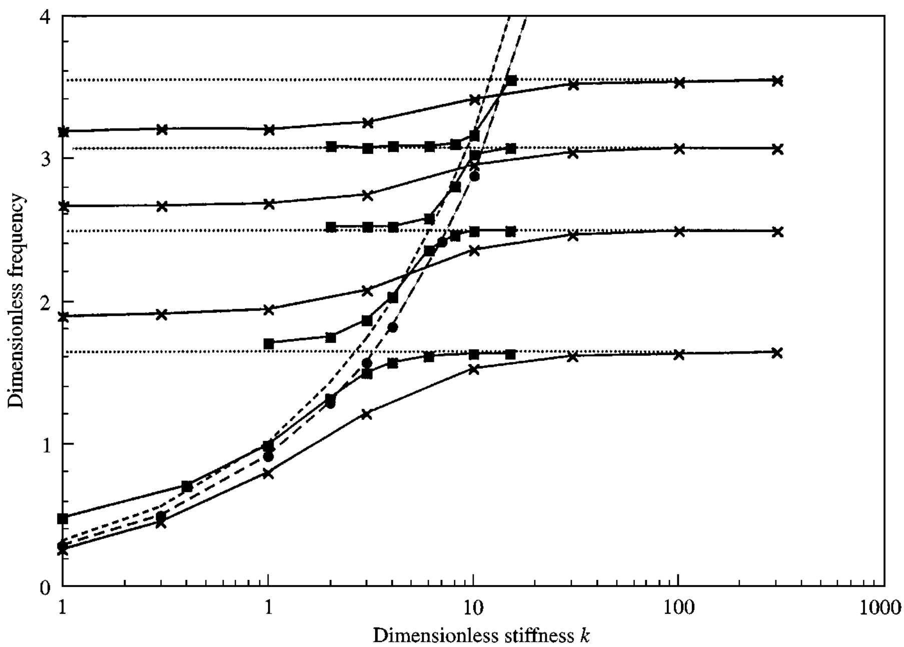

Figure 2. Evolution of the frequencies with stiffness: – – –, $\bar{\omega}_d$ (dry frequency); – – ● – –, $\bar{\omega}_m$ (frequencies with added mass); . . . . , $\bar{\omega}_s$ (sloshing frequency; —■—, $\bar{\omega}_{c\,\mathrm{Chai}}$ (coupled frequencies obtained by Chai et al. (1996)); —×—, $\omega_{c\,\mathrm{Lu}}$ (coupled frequencies obtained by Lu *et al.* (1997).

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